HEAT AND MASS TRANSFER FOR TURBULENT FLOW OF CHEMICALLY REACTING GAS IN ECCENTRIC ANNULAR CHANNELS

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An algorithm is proposed for calculating the velocity, temperature, and concentration fields under conditions of cooling of a cylindrical heat-releasing rod, placed off-center in a circular casing pipe, by a longitudinal flow of chemically reacting gas [N_2O_4].

Annular channels are a widely used structural element in heat exchange equipment and active zones of nuclear reactors. Examples are heat exchangers of the double-pipe type. With the use of annular channels in active zones of nuclear reactors the inner pipe is replaced by a heat-releasing rod with a sheath and a gas gap between them. In practice, because of the structural peculiarities of the channels and inaccuracies of the assembly the longitudinal axes of the heat-releasing rod and of the casing pipe may not be aligned, i.e., an eccentricity appears. In this case in calculating the temperature field not only the nonuniform conditions of cooling at the surface of the rod but also the flow of heat in the rod itself must be taken into account.

An algorithm for calculating heat and mass transfer processes under conditions of flow of a chemically reacting gas N_2O_4 in concentric annular channels is proposed in [1, 2]. However, because of the possibility of using dissociating gases as coolants and working bodies of nuclear power plants [3], it is necessary to develop computational algorithms for calculating heat and mass transfer processes under conditions of nonequilibrium flow of chemically reacting gases not only in axisymmetric channels, but also in channels with a complex transverse cross section (including also in eccentric annular channels).

The transverse cross section of the channel is shown in Fig. 1. The heat-releasing rod with two sheaths is cooled by a longitudinal turbulent flow of coolant, in which the chemical reaction $N_2O_4 \rightleftarrows 2NO_2 \rightleftarrows 2NO + O_2$ occurs. The first stage of the reaction is chemically balanced, while for the second stage it is necessary to take into account the finite rate of the stage. In accordance with the conventional assumptions made for problems of this class, we neglect the flow of heat and mass along the axis of the channel owing to heat conduction and diffusion, thermo- and barodiffusion effects, and secondary flows, and we take into account only the longitudinal component of the velocity. In this case the heat and mass transfer processes, taking into account the anisotropy of the coefficients of turbulent diffusion, are described by the following system of equations:

$$\frac{\partial}{\partial x}\left[(\mu+\rho\varepsilon_x^t)\frac{\partial u}{\partial x}\right]+\frac{\partial}{\partial y}\left[(\mu+\rho\varepsilon_y^t)\frac{\partial u}{\partial y}\right]=\frac{dP}{dz}, \tag{1}$$

$$\int_D \rho u ds=\text{const}, \tag{2}$$

$$\frac{\partial}{\partial x}\left\{\left[\lambda_f\left(1+\frac{\varepsilon_x^t \Pr}{\nu \Pr^t}\right)+\frac{Q_{p\mathrm{I}}}{m_1}\rho D_1\varphi_2\left(1+\frac{\varepsilon_x^t \mathrm{Sc}_1}{\nu \mathrm{Sc}_1^t}\right)\right]\frac{\partial T}{\partial x}\right\}+$$

$$+\frac{\partial}{\partial y}\left\{\left[\lambda_f\left(1+\frac{\varepsilon_y^t \Pr}{\nu \Pr^t}\right)+\frac{Q_{p\mathrm{I}}}{m_1}\rho D_1\varphi_2\left(1+\frac{\varepsilon_y^t \mathrm{Sc}_1}{\nu \mathrm{Sc}_1^t}\right)\right]\frac{\partial T}{\partial y}\right\}=$$

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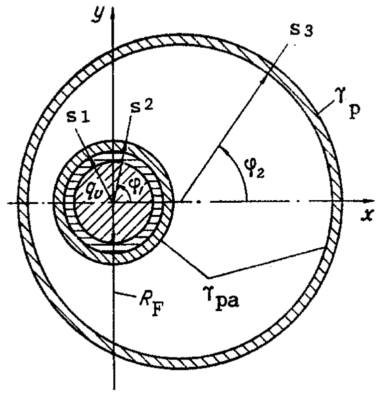

Fig. 1. Transverse section of the annular channel with an eccentricity.

$$= \rho u \left(c_{pf} + \frac{Q_{pI}}{m_1} \varphi_2 \right) \frac{\partial T}{\partial z} - \left(\frac{Q_{pI}}{m_1} \varphi_1 + \frac{Q_{pII}}{m_4} \right) J_4, \tag{3}$$

$$\frac{\partial}{\partial x} \left[\rho D_4 \left(1 + \frac{\varepsilon_x^t \mathrm{Sc}_4^t}{\nu \mathrm{Sc}_4^t} \right) \frac{dC_4}{\partial x} \right] + \frac{\partial}{\partial y} \left[\rho D_4 \left(1 + \frac{\varepsilon_y^t \mathrm{Sc}_4}{\nu \mathrm{Sc}_4^t} \right) \frac{\partial C_4}{\partial y} \right] = \rho u \frac{\partial C_4}{\partial z} - J_4, \tag{4}$$

$$\operatorname{div} \operatorname{grad} T_{\mathrm{F}} + \frac{q_{v\,\mathrm{F}}}{\lambda_{\mathrm{F}}} = 0, \tag{5}$$

$$\operatorname{div} \operatorname{grad} T_{\mathrm{s}i} = 0, \quad i = 1,\ 2,\ 3, \tag{6}$$

with the boundary conditions:

$$\lambda_{\mathrm{F}} \frac{\partial T_{\mathrm{F}}}{\partial n} = \lambda_{\mathrm{s1}} \frac{\partial T_{\mathrm{s1}}}{\partial n} \Big|_{\gamma_1}, \quad T_{\mathrm{F}} = T_{\mathrm{s1}}, \tag{7}$$

$$\lambda_{\mathrm{s1}} \frac{\partial T_{\mathrm{s1}}}{\partial n} = \lambda_{\mathrm{s2}} \frac{\partial T_{\mathrm{s2}}}{\partial n} \Big|_{\gamma_2}, \quad T_{\mathrm{s1}} = T_{\mathrm{s2}}, \tag{8}$$

$$\lambda_{\mathrm{s}i} \frac{\partial T_{\mathrm{s}i}}{\partial n} = \left(\lambda_f + \frac{Q_{pI}}{m_1} \varphi_2 \rho D_1 \right) \frac{\partial T}{\partial n} \Big|_{\gamma\mathrm{pa}}, \quad i = 2,\ 3,\ T_{\mathrm{s}i} = T, \tag{9}$$

$$u = \frac{\partial C_4}{\partial n} = 0 \,|_{\gamma_{\mathrm{pa}}}, \tag{10}$$

$$\frac{\partial T}{\partial n} = 0 \,|_{\gamma_{\mathrm{p}}}, \tag{11}$$

where γ_1 and γ_2 are the boundary between the rod and first sheath and the boundary between the sheaths, respectively; γ_{pa}, surface of the annular channel; γ_p, outer surface of the casing pipe; the values of the index i = 1 and 2 refer to the first and second sheaths of the heat-releasing rod, and i = 3 refers to the casing pipe.

The coefficients of turbulent diffusion were calculated using the procedure proposed in [4]:

$$\begin{aligned} \varepsilon_x^t &= 0{,}18 f_0(\eta) f_1(\eta) L_x^2 \left| \frac{\partial u}{\partial n} \right|, \\ \varepsilon_y^t &= 0{,}18 f_0(\eta) f_1(\eta) L_y^2 \left| \frac{\partial u}{\partial n} \right|. \end{aligned} \tag{12}$$

Here L_x and L_y are the "directed" scales, calculated for rectilinear channels using the formulas

$$\frac{1}{L_x}=\frac{2}{\pi}\int_0^{2\pi}\frac{1}{l}|\cos(\varphi,\ x)|\,d\varphi;\quad \frac{1}{L_y}=\frac{2}{\pi}\int_0^{2\pi}\frac{1}{l}|\cos(\varphi,\ y)|\,d\varphi; \tag{13}$$

$$f_0(\eta)=\exp(-\eta);\ f_1(\eta)=\frac{1}{\eta}(1-\exp(-\eta)),\ \eta=\frac{65}{\gamma_*},\ \gamma_*=\frac{L^2}{\nu}\left|\frac{du}{dn}\right|;$$

where L is the "isotropic" scale, which for rectilinear channels can be determined from the formula

$$\frac{1}{L}=\frac{1}{2}\int_0^{2\pi}\frac{1}{l}\,d\varphi,$$

where $\ell(\phi)$ is the distance from the point under study to the channel wall in the direction ϕ.

We shall replace the derivatives along the channel in Eqs. (1), (3), and (4) by finite-difference analogs. In addition, the equation of motion was integrated with the use of a purely implicit scheme (Euler's scheme), while the energy and diffusion equations were integrated using the Crank–Nicholson scheme. After the difference approximation of the derivatives of the quantity sought along the axis of the channel, the solution of Eqs. (1), (3), and (4) can be sought as a minimum of the functionals [5]:

$$\Phi(u)=\frac{1}{2}\int_D[F_1(\mathrm{grad}\,u)^2+P_1u^2]\,ds-\int_D M_1u\,ds, \tag{14}$$

$$\Phi(T)=\frac{1}{2}\int_D[F_2(\mathrm{grad}\,T)^2+P_2T^2]\,ds-\int_D M_2T\,ds-\int_{\gamma_{\mathrm{pa}}}F_2T\frac{\partial T}{\partial n}\,d\gamma, \tag{15}$$

$$\Phi(C_4)=\frac{1}{2}\int_D[F_3(\mathrm{grad}\,C_4)^2+P_3C_4^2]\,ds-\int_D M_3C_4\,ds. \tag{16}$$

The pressure gradient dP/dz is found from the condition that the flow rate along the channel is constant. Using the general solutions for the equations of heat conduction for a rod, the sheaths, and the casing pipe in the form of Fourier series with unkown coefficients, following [5] we write the functional for the temperature as follows:

$$\Phi(T)=\frac{1}{2}\int_D[F_2(\mathrm{grad}\,T)^2+P_2T^2]\,ds-\int_D M_2T\,ds-\frac{q_VR_{\mathrm{F}}^2}{2}\int_0^{\pi}T(R_{\mathrm{s2}})\,d\varphi+$$
$$+\frac{\lambda_{\mathrm{s2}}}{\pi}\sum_{k=1}^{\infty}k\beta_k\left(\int_0^{\pi}T(R_{\mathrm{s2}})\cos k\varphi\,d\varphi\right)^2+\frac{\lambda_{\mathrm{s3}}}{\pi}\sum_{k=1}^{\infty}k\left(\int_0^{\pi}T(R_{\mathrm{s3}})\cos k\varphi\,d\varphi\right)^2. \tag{17}$$

To determine the minima of the functionals (14), (16), and (17) we shall employ the method of finite elements [6]. After the corresponding transformations the problem reduces to the solution of systems of algebraic equations for the parameters at the nodal points u_p, T_p, C_{4p}:

$$\sum_{p=1}^{n}(A_{jp}^{(1)}+B_{jp}^{(1)})u_p=B_j^{(1)},\ \ j=1,\ \ldots,\ n;\ p=1,\ \ldots,\ n, \tag{18}$$

$$\sum_{p=1}^{n}(A_{jp}^{(2)}+P_{jp}^{(2)}+J_{jp}^{(2)})T_p=B_j^{(2)}+C_j^{(2)},\ \ j=1,\ \ldots,\ n;\ p=1,\ \ldots,\ n, \tag{19}$$

$$\sum_{p=1}^{n}(A_{jp}^{(3)}+P_{jp}^{(3)})C_{4p}=B_j^{(3)},\ \ j=1,\ \ldots,\ n;\ \ p=1,\ \ldots,\ n. \tag{20}$$

The matrix $J^{(2)}$ is a component of the stiffness matrix, taking into account the heat conduction processes in the fuel core, the sheaths, and the casing pipe. Thus the determination of the velocity, temperature, and concentration fields reduces to solving the algebraic equations (18)-(19) at each step along the channel.

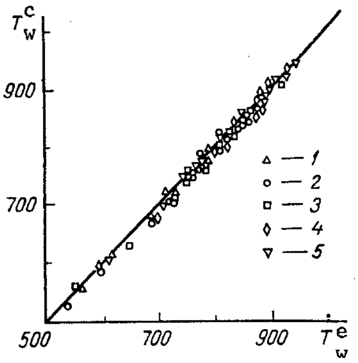

Fig. 2

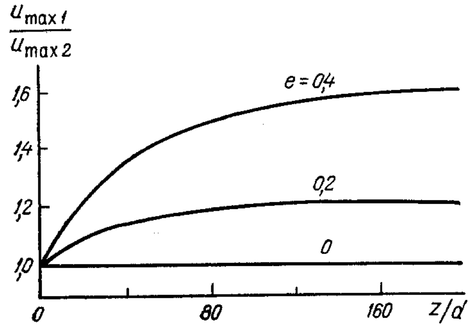

Fig. 3

Fig. 2. Comparison of experimental [7] and computed wall temperatures of a concentric annular channel for different cross sections at the channel inlet: 1) z = 0.065 m; 2) 0.13; 3) 0.26; 4) 0.39; 5) 0.45.

Fig. 3. Change in the ratio of the maximum velocity u_{max1} at $\phi_1 = 0°$ to the maximum velocity u_{max2} at $\phi_1 = 180°$ along the channel for different eccentricities.

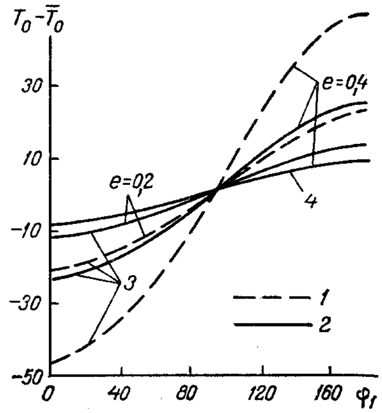

Fig. 4. Temperature distribution along the surface of the rod (P_{in} = 75 bar, T_{in} = 590 K, $C_{4in} = C_{4e}$ (P_{in}, T_{in}), G = $1.6 \cdot 10^{-2}$ kg/sec, $\bar{q}_c = 6.2 \cdot 10^5$ W/m², $R_F = 2.6 \cdot 10^{-3}$ m, $R_{s1} = 2.7 \cdot 10^{-3}$ m, $R_{s2} = 3.1 \cdot 10^{-3}$ m, $R_{s3} = 3.7 \cdot 10^{-3}$ m); 1) "frozen" flow and 2) chemically reacting flow.

As established as a result of a numerical experiment, in integrating the energy equation there arise oscillations of the temperature, whose magnitude decreases as the sizes of the elements and the size of the integration step Δz along the channel axis decrease. Therefore at each step along the channel axis the energy equation was solved twice: first, using the Crank—Nicholson scheme, the temperature was determined at the point i + 1/2 as $(T_i + T_{i+1})/2$, and then with the help of the same scheme the temperature at the i + 1-st step was found as $(T_{i+1/2} + T_{i+3/2})/2$. In this approach the temperature oscillations arising in the course of the numerical solution could be completely avoided.

To substantiate the reliability of the computational results obtained using the proposed algorithm we calculated a number of operating regimes [7], in which heat transfer under conditions of turbulent flow of the dissociating coolant nitrine in concentric annular channels was studied. Figure 2 shows the distribution of the computed T_w^c and experimental T_w^e wall temperatures for different cross sections at the inlet to the channel. The deviation of the computed temperatures from the experimental temperatures falls in the range ±15°K.

Figure 3 shows the ratio of the maximum velocities in the wide (u_{max1}) and narrow (u_{max2}) parts of the channel at different distances from the inlet section. One can see that increasing the relative eccentricity increases the section in which the value of u_{max1}/u_{max2} is stabilized.

Figure 4 shows the temperature distribution along the surface of the heat-releasing rod (z = 0.35 m) for a chemically reacting flow for different values of the relative eccentricity and coefficients of thermal conductivity of the fuel, sheaths, and casing pipe. The curves 3 correspond to the case λ_F = 16 W/(m·deg), λ_{s1} = 0.4 W/(m·deg), λ_{s2} = 20 W/(m·deg), λ_{s3} = 20 W/(m·deg), while curve 4 corresponds to the case λ_F = 64 W/(m·deg), λ_{s1} = 1.6 W/(m·deg), λ_{s2} = 80 W/(m·deg), λ_{s3} = 80 W/(m·deg). The figure also compares the results of calculations of the corresponding variants of the "frozen" flow. One can see from the figure that chemical rections in the flow reduce the nonuniformity of the temperature along the perimeter of the sheath of the rod compared with the case of the "frozen" flow. This is caused by heat transfer owing to concentration-driven diffusion. In analyzing the dependence of the nonuniformity of the surface temperature of the rod on the coefficients of thermal conductivity and the geometric dimensions of the fuel core and of the sheaths it could be useful to employ the generalized thermal similarity parameter γ_k [8]. This parameter for a rod with two sheaths has the form

$$\gamma_k = \frac{\lambda_{s_2}}{\lambda_f}\,\frac{x_1(y_2 - x_2)\,y_1 + x_2(1 - x_2 y_2)}{x_1(y_2 + x_2)\,y_1 + x_2(1 + x_2 y_2)},$$

where $y_1 = (\lambda_{s1} - \lambda_F)/(\lambda_{s1} + \lambda_F)$; $y_2 = (\lambda_{s2} - \lambda_{s1})/(\lambda_{s2} + \lambda_{s1})$; $x_1 = (R_F/R_{s2})^{2k}$; $x_2 = (R_{s1}/R_{s2})^{2k}$.

If $\gamma_k \to \infty$, then boundary conditions of the first kind (T_w = const) are realized on the surface of the rod. One can see from Fig. 4 that when the coefficients of thermal conductivity of the fuel and sheath materials are quadrupled (which corresponds to quadrupling γ_k also) the temperature nonuniformity on the surface of the sheath decreases by a factor of ∿3.

NOTATION

u, velocity; μ, coefficient of dynamic viscosity; ρ, density; c_p, heat capacity; D, coefficient of diffusion; m_k, molecular mass of the k-th component; ε, coefficient of turbulent viscosity; G, flow rate; s, area; q_V, volume liberation of heat; λ, coefficient of thermal conductivity; K_c, reaction rate constant; Q_p, heat released in the reaction; e, relative eccentricity; and z, longitudinal coordinate. Indices: I and II, reactions $N_2O_4 \rightleftarrows 2NO_2$ and $2NO_2 \rightleftarrows 2NO + O_2$, respectively; 1) N_2O_4; 2) NO_2; 3) NO; 4) O_2; f, fuel; s, sheath; "in", inlet section; e, equilibrium value; and t, turbulent value.

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